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A PROCEDURE FOR DETERMINING ELASTIC MODULI OF SOILS BY FIELD VIBRATORY TECHNIQUES

MISCELLANEOUS PAPER NO. 4-577

June 1963

U. S. Army Engineer Waterways Experiment Station
CORPS OF ENGINEERS
Vicksburg, Mississippi

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Preface

The study reported herein comprises a portion of the study of foundation behavior under dynamic loadings which is being performed for the Office, Chief of Engineers, by the U. S. Army Engineer Waterways Experiment Station in accordance with "Instructions and Outline, Development and Evaluation of Soil Bearing Capacity, FY 1961-62-63." The proposed procedure for determination of elastic moduli by vibratory techniques is the result of field tests performed by the WES at numerous locations during 1960-1962.

Engineers of the Flexible Pavement Branch, WES, actively engaged in the data collection, analysis, and report phases of this study were Messrs. A. A. Maxwell, O. B. Ray, Z. B. Fry, A. H. Joseph, and R. F. Ballard, Jr. The work was under the general supervision of Mr. W. J. Turnbull, Chief, Soils Division. This report was prepared by Mr. Fry.

The panel of consultants who monitored the program comprised Professors N. M. Newmark, R. B. Peck, and W. J. Hall, University of Illinois; Prof. F. E. Richart, University of Michigan; Prof. R. V. Whitman, Massachusetts Institute of Technology; Prof. R. E. Fadum, North Carolina State College; and Mr. S. D. Wilson, Shannon and Wilson, Seattle, Washington.

Directors of the WES during the data collection and preparation of this report were Col. Edmund H. Lang, CE, and Col. Alex G. Sutton, Jr., CE. Technical Director was Mr. J. B. Tiffany.

Contents

List of Symbols

D Depth, ft

E Young's modulus of elasticity, psi

f Frequency, cps

g Acceleration due to gravity, 32.2 ft/sec/sec

G Modulus of shear elasticity, psi

V Velocity of propagated wave, fps

V_c Velocity of compression wave, fps

V_s Velocity of shear wave, fps

X Slope intersection distance

γ Wet density of soil, lb/cu ft

λ Wavelength, ft

μ Poisson's ratio

ρ Mass density of soil, $\frac{\gamma}{g}$

A PROCEDURE FOR DETERMINING ELASTIC MODULI OF SOILS BY FIELD VIBRATORY TECHNIQUES

Background, Purpose, and Scope

1. Federal agencies engaged in planning facilities for launching or operating spacecraft, heavy weapons, or delicate electronic equipment require accurate and quick information on a soil's ability to resist deformation over a considerable area for use in site selection and foundation design. Field and laboratory tests and analytical studies have been conducted to develop equipment and procedures for determining the elastic moduli (and thus the resistance to deformation) of foundation soils by vibratory techniques. This report describes the equipment, measuring techniques, and method of computing the elastic moduli, as developed to date. Since these techniques are relatively new, and the method of computation is still in the development stage, studies to refine them are continuing and results will be published as they become available.

2. The method and procedure described herein were developed through the work of many consultants and specialists. Much of the basic theory and analytical work was performed by the Royal Dutch/Shell Laboratorium, Amsterdam, Holland. The equipment constructed in the early stages of development of the method has been improved through field usage and experience.

3. The method employs the measurement of the velocity of waves propagated at known frequencies along the exposed surface of the soil. Determining the velocity of wave propagation over a range of frequencies provides a reliable means of deriving the elastic constants of a soil, and ultimately provides an estimate of its ability to resist deformation. The method has an advantage over conventional soil tests in that it is applied to soil in situ and enables a considerable area to be tested rapidly. The principle employed has shown great promise, primarily due to the excellent correlation and validation obtained by means of independent laboratory dynamic tests. In the determination of the elastic moduli of a soil by use of vibratory techniques, a basic understanding of dynamics is helpful but not essential.

Basic Theory

Wave propagation

4. When sustained vibrations are induced into a soil, concentric waves are propagated outward from the source. The waves require a time $\frac{X}{V}$ to travel a distance X through the soil in which the wave velocity is V. If the waves are propagated at a known frequency f, then

$$V = \lambda f \tag{1}$$

where

λ = wavelength, ft

This velocity is dependent upon the ratio of the elasticity of the medium to its mass density ρ, and the wave type. If the shear modulus G is taken as a measure of the elastic properties, the shear wave velocity V_s is defined by:

$$V_s = \sqrt{\frac{Gg}{\gamma}} = \sqrt{\frac{G}{\rho}} \quad \text{(see reference 4)} \tag{2}$$

where

γ = wet density of soil, lb per cu ft

ρ = mass density = $\frac{\gamma}{g}$

g = acceleration due to gravity = 32.2 ft/sec/sec

5. In regard to the wave type, R. Jones[5*] believes that Rayleigh (surface) waves are the most probable, whereas Heiland[2] thinks it fairly certain that transverse waves are the most probable, and it is the opinion of A. Ramspeck[2] that Love waves are the most probable. Richart and others,[7] citing Miller and Percy (1955), state that in an elastic solid with a Poisson's ratio of 0.25 for the case of a single source of vertical load on a free surface, 67 percent of the energy is dissipated as Rayleigh waves, 26 percent as shear waves, and only 7 percent as compression waves. While the ground does not behave in a purely elastic manner, this does indicate that the predominant wave would be expected to be a Rayleigh (surface)

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* Raised numerals refer to similarly numbered items in list of references at end of text.

wave. However, the differences between velocities of Rayleigh and shear waves with changes in Poisson's ratio are so small as to be of no practical significance in connection with foundation problems. A relation between Poisson's ratio and velocities of propagation of compression, shear, and Rayleigh waves is shown in fig. 1. Since, as shown by fig. 1, the velocities of Rayleigh and shear waves are so similar, the propagated surface waves can be considered to be shear waves for all practical purposes and their velocities can be determined by vibratory tests.

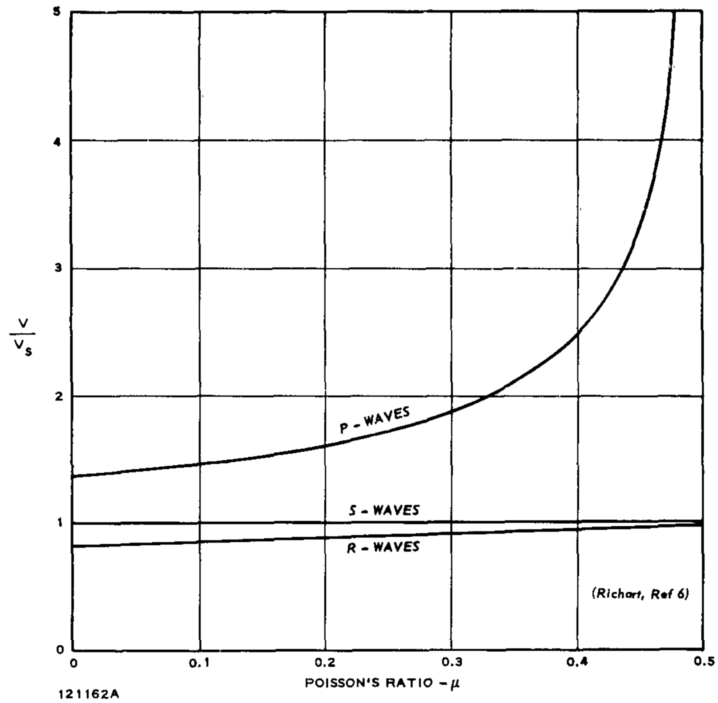

Fig. 1. Relation between Poisson's ratio (μ), and velocities of propagation of compression (P), shear (S), and Rayleigh (R) waves in a semi-infinite elastic medium

<u>Poisson's ratio</u>

6. Poisson's ratio μ of the soil can be determined by the relation of shear wave velocities V_s and compression wave velocities V_c. The shear wave velocities are determined by vibratory tests, and the compression wave velocities by ordinary seismic refraction measurements.* The relation is

$$\mu = \frac{1 - 2\left(\frac{V_s}{V_c}\right)^2}{2 - 2\left(\frac{V_s}{V_c}\right)^2} \tag{3}$$

* Preferably, the velocities of both shear and compression waves should be determined by the same method. The error, if any, introduced by the use of different methods is being investigated.

Elastic moduli

7. The shear modulus G is related to Young's modulus E and Poisson's ratio μ of the soil by

$$E = 2(1 + \mu)\, G \quad \text{(see reference 3)} \tag{4}$$

or it can be derived that

$$E = 2(1 + \mu)\, V_s^2\, \rho \tag{5}$$

and

$$G = V_s^2\, \rho \tag{6}$$

or

$$G = \frac{E}{2(1 + \mu)} \tag{7}$$

It should be noted here that equations 2 through 7 are for homogeneous, isotropic, elastic materials. From these equations, values of E and G can be determined through the measurement of shear and compression wave velocities, provided the density of the soil is known. Actually the density of the soil is a minor consideration as it shows little variation as compared with the shear wave velocity which may vary between 300 and 1200 fps, so that V^2 displays a considerable variation for various soils.

Equipment and Measuring Techniques

Vibratory test equipment

8. Vibrators. In inducing vibrations in the soil, different vibrators are used for high- and low-frequency ranges. For high frequencies, 30 to 10,000 cps, an electromagnetic vibrator is used. For low frequencies, below 30 cps, a heavier mechanical vibrator is used. The high-frequency vibrator has a force output of 50 lb at 500 cps and power of 100 w. This force output suffices for the high frequencies, as the resulting waves remain near the surface and only a small part of the soil has to be set in motion. The larger mechanical vibrator consists of counterrotating eccentric weights that produce forces of approximately 100 lb at 5 cps and 3500 lb at 30 cps.

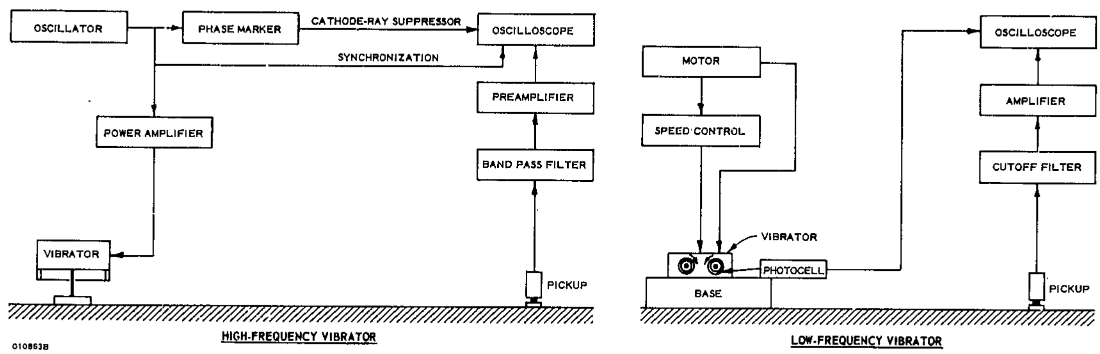

Fig. 2. Block diagrams for vibrators and associated electronic equipment

9. Auxiliary equipment. The auxiliary equipment needed with the vibrators is shown in fig. 2, and a detailed list of makes and model numbers is given below. The equipment is primarily the same for both vibrators except for that used in driving them; also the test procedure is essentially the same for both vibrators except in producing the break in the sine wave display on the oscilloscope. These differences will be described in the following subparagraphs.

Item	Make and Model
	High-Frequency Vibrator
Vibrator	Goodman 390A
Electric generator	Kohler 5 KVA
Oscilloscope	Hewlitt-Packard 130 BR
Oscillator	Hewlitt-Packard 202 CR
Amplifier	Sovereign 120 watt
Variable filter	Allison 2 ABR
Velocity pickup	Phillips GM 5520 or PR 9260
Phase marker	WES-constructed
	Low-Frequency Vibrator
Vibrator	WES-constructed counterrotating eccentric weights
Hydraulic transmission power	New York Air Brake Company "Dynapower"
Cutoff filter	Allison
Oscilloscope	Hewlitt-Packard 130 BR
Amplifier	WES-constructed
Velocity pickup	Phillips GM 5520 or PR 9260
Phase marker	WES-constructed photocell

a. High-frequency equipment. The electromagnetic vibrator is powered by a 120-w power amplifier, and the frequency is

accurately controlled by a standard R. C. oscillator. The sinusoidal wave produced by the vibrator is received by a velocity-type pickup and transmitted through a band pass filter to eliminate undesirable noise. The signal is then preamplified before appearing on the screen of the oscilloscope in a sine form display. A phase mark, whose frequency is also controlled by the oscillator driving the vibrator, is added to the signal on the oscilloscope. The mark is obtained by cathode-ray suppression on the z-axis of the oscilloscope so that there is a short interruption of the sine wave display as it appears on the oscilloscope. The combination of the phase mark and sine wave displayed on the oscilloscope appears as a sine wave with a break or spike, as shown in fig. 3.

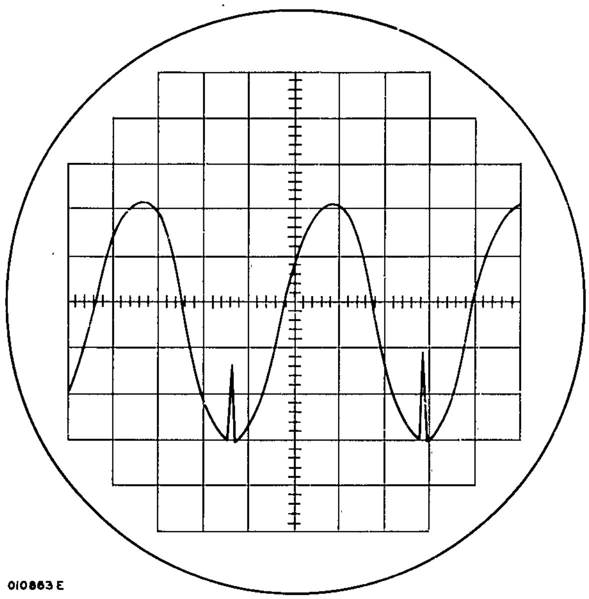

Fig. 3. Typical display of sine wave on screen of oscilloscope showing phase mark in trough

b. Low-frequency equipment. The larger mechanical vibrator is driven by a constant-displacement hydraulic motor and belt-pulley arrangement with frequency controlled by a variable-displacement hydraulic pump driven by a gasoline engine. The vibrator is fitted with a device which measures the phase angle between the position of the eccentric weight when actual displacement occurs and the vertical position of the weight at rest. The device consists of a movable ring, graduated in degrees, attached to a shaft guard. A counterbalanced arm is affixed to the center of the shaft of the weight in such a manner that the arm points downward covering a photocell mounted in the movable ring. An accelerometer or velocity-type pickup can be used to receive the sinusoidal

output of the vibrator. The signal received by the pickup is channeled to a 35-cps cutoff filter, then through an amplifier to an oscilloscope. Simultaneously, the voltage spike, induced by the photocell, as the counterbalanced arm rotates and obstructs the light received by the cell, is also displayed on the oscilloscope screen. The resulting spike is superimposed on the sine wave and can be moved transversely across the screen by turning the movable ring that houses the photocell. The resulting display on the oscilloscope screen is similar to that shown in fig. 3.

Vibratory test technique

10. The vibrator is positioned on the surface of the soil and set in operation at a constant frequency. A measuring tape is extended from the vibrator in the direction in which measurements are desired. The pickup is then placed on the ground surface as near the vibrator as possible. The sine wave and phase mark are displayed on the screen of the oscilloscope. The phase mark is then manipulated into a trough or peak of the sine wave and not disturbed until measurements at a set frequency have been completed. When these adjustments have been made, the pickup is moved away from the vibrator along the line of the measuring tape. As the pickup is moved, the signal will show a steadily increasing phase shift between the sine wave and phase mark. As this shift increases, the phase mark will coincide with the succeeding trough or peak of the sine wave. When this occurs, the distance indicated on the tape should be recorded, as shown in plate 1. The pickup is then moved farther away from the vibrator along the tape, and each distance where the phase mark coincides with a trough or peak in the sine is recorded. This is continued until a sufficient number of wavelengths, preferably four to eight or more, have been obtained. The frequency of the vibrator is then changed, usually increased, and the procedure is repeated.

Seismic equipment and technique

11. The basic function of the refraction seismograph is to measure the time required for a compression wave to travel through the soil. The wave may be produced by detonating an explosive charge set in the ground[8] or by striking with a hammer a steel plate embedded in the ground.[1] The latter method is being used more and more due to its compactness and portability as well as accuracy. The equipment consists of a sledgehammer instrumented with an impact switch as a source of impulse, and a geophone

embedded in the soil as a pickup station. The hammer is used to strike a blow on a striking plate which is placed at regular known intervals away from the geophone. Upon impact, the switch on the hammer closes, setting a binary counter into operation. When the compression wave produced by the hammer blow reaches the geophone, the counter is automatically stopped; this indicates the elapsed time the wave required to travel from the point of impact to the geophone. These data can be plotted in graphic form as distance versus travel time, as shown in plate 2. The reciprocal of the slope of the lines indicates the velocity of the wave through each medium encountered. A change in slope of the line indicates that the wave passed from a low-velocity medium to a higher velocity medium at a depth D which can be calculated from the relation

$$D = \frac{X}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}} \quad \text{(see reference 1)} \qquad (8)$$

where

X = slope intersection distance (i.e. distance, from point of impact, at which the slopes intersect)

V_1 = velocity of the medium in which the first slope was produced

V_2 = velocity of that in which the second slope was produced

The compression wave velocity of each medium is used in conjunction with the shear wave velocity for the corresponding depth of the medium to compute a Poisson's ratio (equation 3) for that material.

Method of Computation

12. The basic data obtained by means of the vibratory technique are recorded as shown in plate 1. These data are then plotted as distances between troughs and peaks as a function of the number of wavelengths, as shown in plate 3. The wavelength for a particular frequency is determined as the reciprocal of the slope of the line through the plotted points. The velocity of the shear wave is then determined by equation 1. A compression wave velocity is determined in a similar manner (plate 2) and used together with the shear wave velocity to determine a Poisson's ratio by equation 3. Caution should be exercised to ensure selection of the shear

and compression wave velocities for corresponding media or depths. As an approximation, it can be said that propagation of the shear wave takes place at a depth equal to about one-half the wavelength. Therefore, using one-half the wavelength as the depth at which the velocity of propagation of the shear wave occurs and the velocity of the compression wave at a corresponding depth, a Poisson's ratio can be obtained for materials at different depths. The elastic moduli, E and G, can then be computed by means of equations 5 and 7, respectively. As shown in plate 4, it is usually convenient to plot E and G versus depth, which again is considered to be one-half the wavelength of the shear wave. Such a plot provides a visual picture of the change in soil characteristics with increasing depth. Should very significant differences of E and G occur with depth, the data can be represented as step-type curves as shown in plate 4. If the differences are not so significant, the data are best represented by a smooth curve indicating the gradual increase in E and G with depth, as shown in plate 5. The depth can also be expressed as overburden pressure in pounds per square inch, providing a comparison of E and G with increasing pressure.

Summation

13. This report has presented a procedure by which rapid determination of the elastic characteristics of in-situ soil materials can be made. Using the proposed methods, the shear and compression wave velocities and Poisson's ratio can be determined, and from these the shear, G, and compression, E, moduli can be computed. These characteristics may then be used in the design and/or evaluation of foundations for structures.

References

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TYPICAL
VELOCITY DETERMINATIONS

LOCATION: Test #1

DATE: 24 March 1962

NO. OF WAVE LENGTHS	FREQUENCY IN CPS AT INDICATED DISTANCE IN FT						
	60	75	90	150	225	325	
0.0	0.3	0.3	0.3	0.3	0.3	0.3	
0.5	6.0	3.4	2.5	1.15	1.1	0.8	
1.0	12.5	7.6	5.2	2.9	1.8	1.35	
1.5	19.7	11.5	8.2	4.3	2.5	1.9	
2.0	22.0	15.2	11.1	5.7	3.5	2.4	
2.5	28.5	18.9	13.9	7.3	4.45	3.15	
3.0	34.3	22.7	16.6	8.8	5.4	3.7	
3.5	44.0	27.4	19.5	10.35	6.6	4.35	
4.0			21.9	11.9	7.5	—	
4.5				13.2		5.6	
5.0							
5.5							
6.0							
6.5							
7.0							
7.5							
8.0							
8.5							
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10.0							
10.5							
11.0							
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14.5							
15.0							

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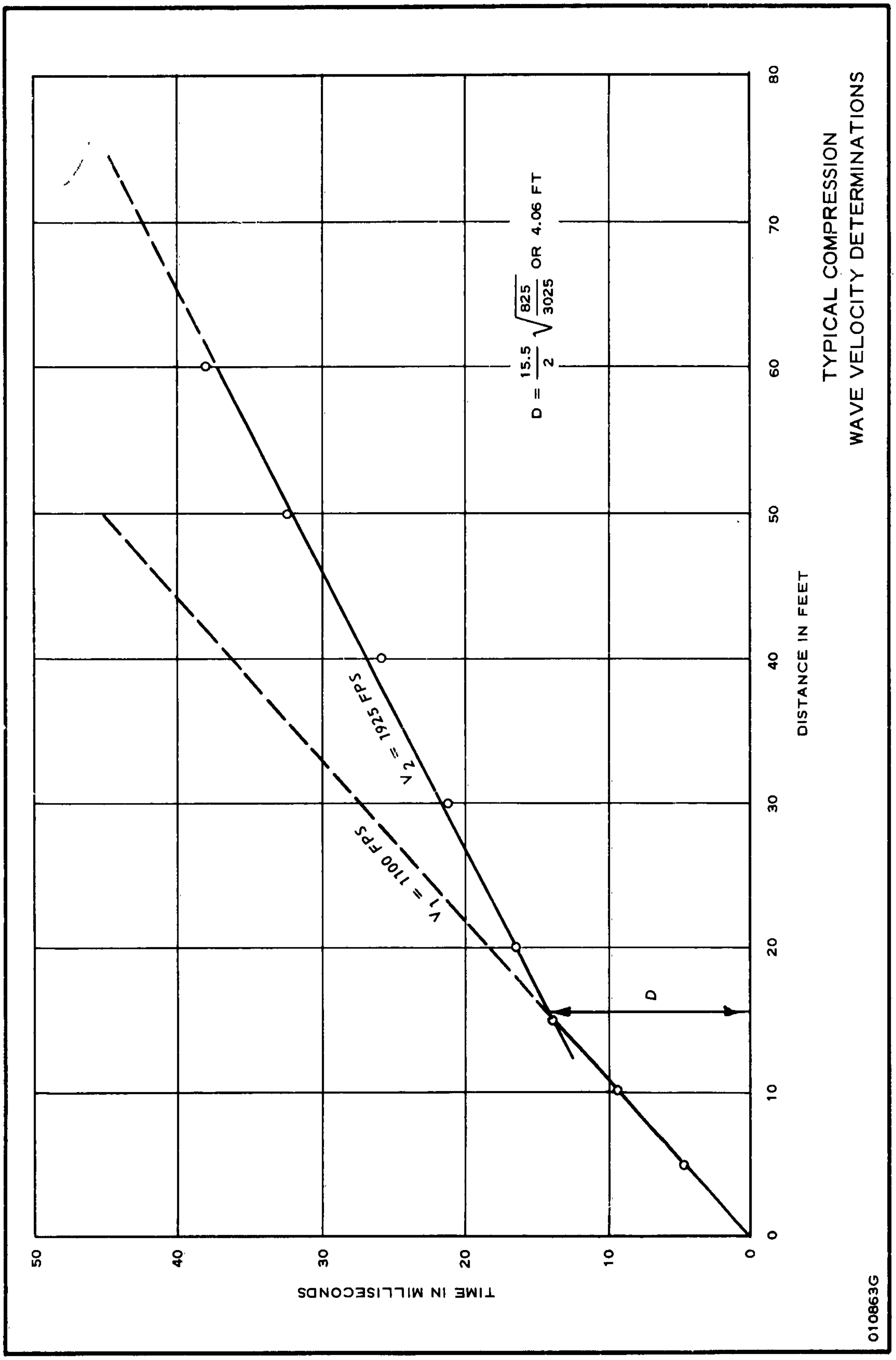
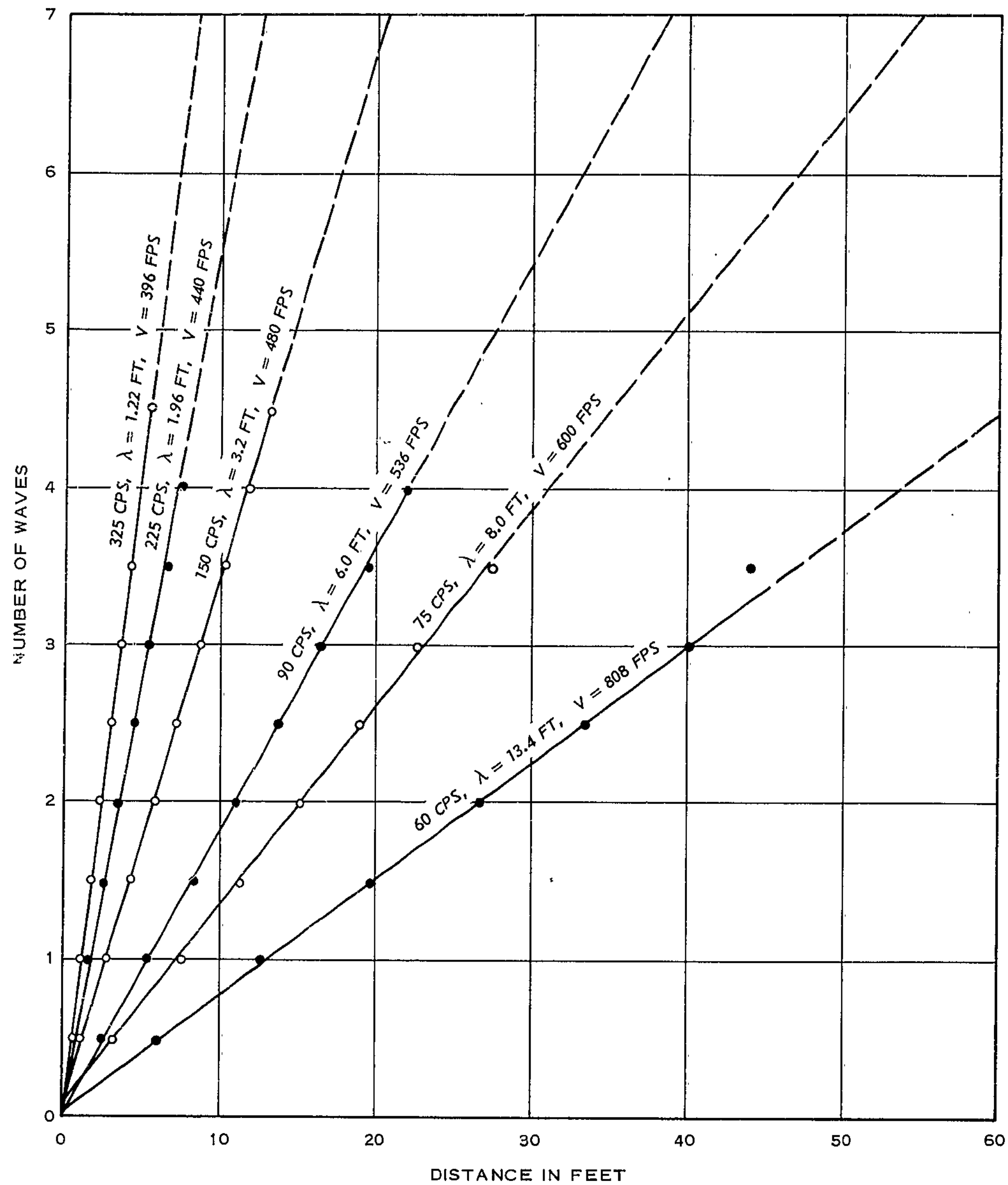

TYPICAL COMPRESSION
WAVE VELOCITY DETERMINATIONS
$D = \frac{15.5}{2}\sqrt{\frac{825}{3025}}$ OR 4.06 FT
V_2 = 1925 FPS
V_1 = 1100 FPS
D
DISTANCE IN FEET
TIME IN MILLISECONDS
0
10
20
30
40
50
60
70
80
010863G

PLATE 2



TYPICAL SHEAR WAVE
VELOCITY DETERMINATIONS

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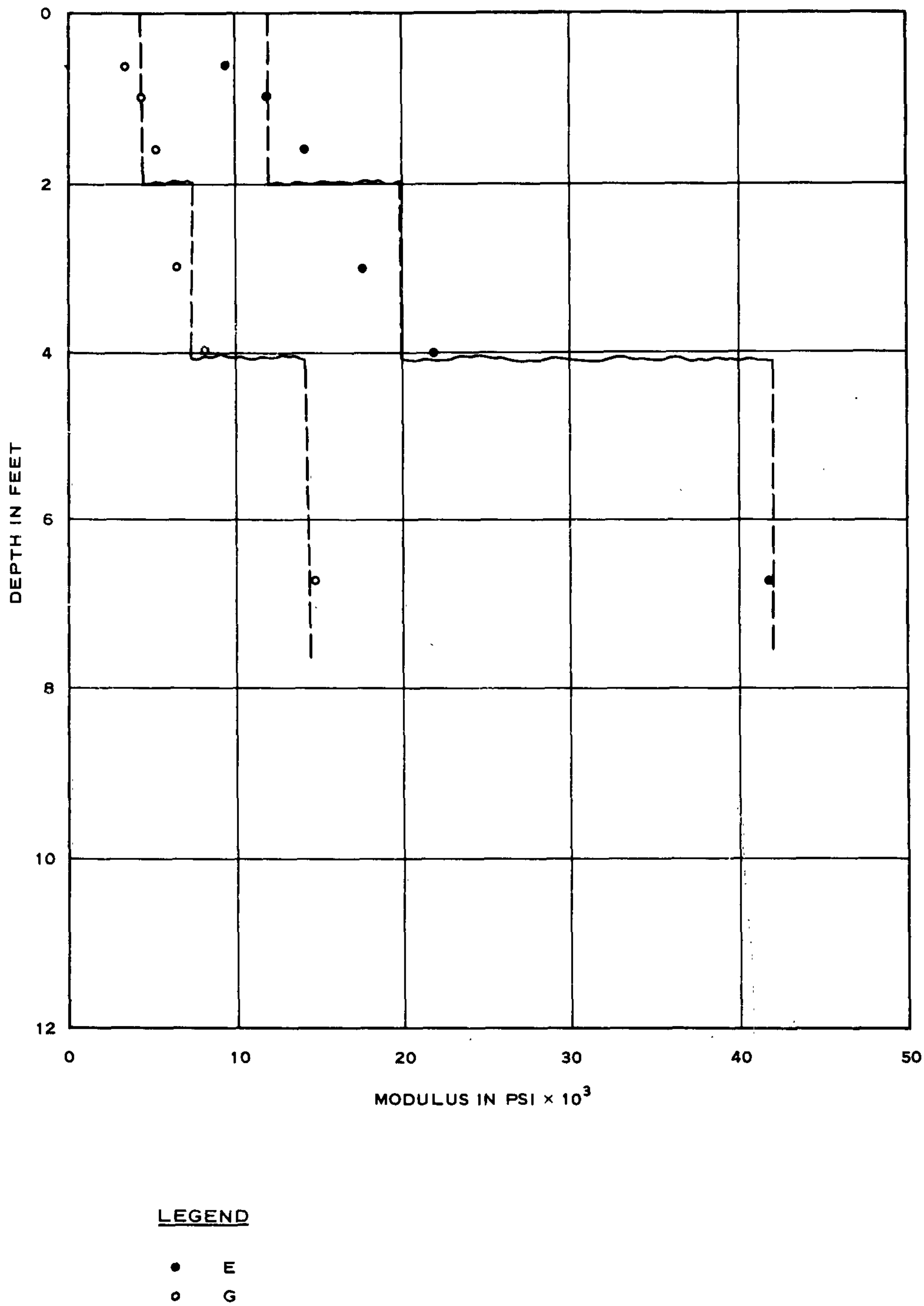
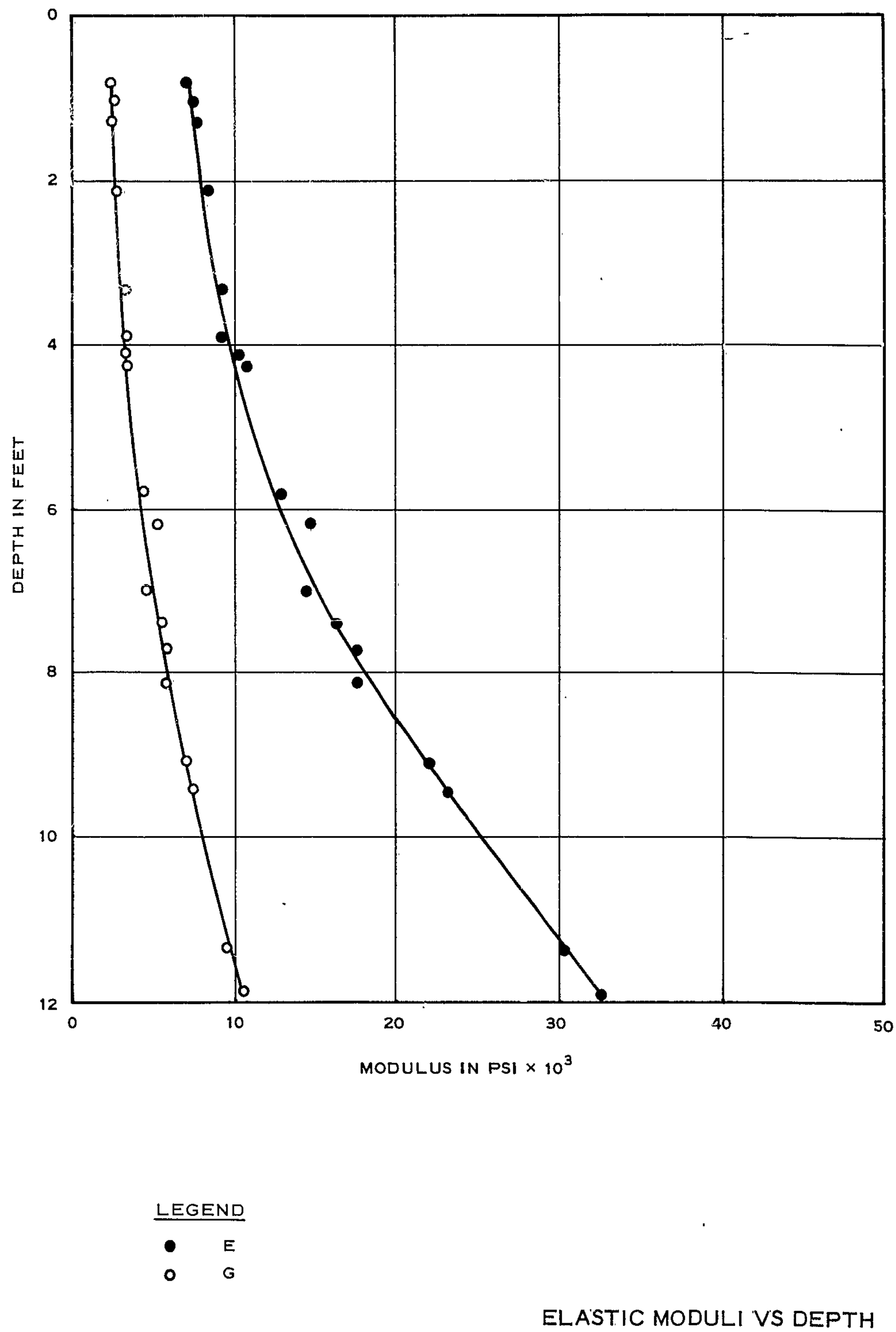


ELASTIC MODULI VS DEPTH

PLOTTED AS STEP-TYPE CURVES

010863I

PLATE 4



ELASTIC MODULI VS DEPTH

PLOTTED AS SMOOTH CURVES

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